

Review of Exponents

What are Exponents?

Exponents are a shorthand way to show how many times a number (called the base) is multiplied to itself. For example, in the **power** 8^2 , 8 is the **base** and 2 is the **exponent**. This expression tells us to multiply 8 to itself twice, or “8 to the power of 2”.

$$8^2 = (8 \times 8) = 64$$

Warning: Watch for the Karat! Sometimes we use the karat symbol \wedge (Shift-6 on a computer keyboard) to indicate an exponent. For example: $2\wedge 4 = 2^4 = 2 \times 2 \times 2 \times 2 = 16$.

Negative Exponents

In mathematics, the negative symbol actually means “the opposite of”. The opposite of a positive number is a negative number. The opposite of a negative number is a positive number ($-(-x) = x$). The opposite of addition is subtraction. The opposite of multiplication is division.

Because exponents mean multiple-multiplication, the opposite of an exponent means multiple-division. To turn a negative exponent into a positive exponent, simply move the power to the other side of the division line, and make the exponent positive.

$$\frac{a^3 b^{-5}}{c^{-4} d^7} = \frac{a^3 c^4}{b^5 d^7}$$
$$\frac{3}{x^{-4}} = 3x^4$$

Example 1: Moving a negative power in the numerator to the denominator.

$$8^{-1} = \frac{1}{8^1} = \frac{1}{8} = 1 \div 8 = 0.125$$

Example 2: Moving a negative power in the denominator to the numerator.

$$\frac{1}{12^{-3}} = \frac{12^3}{1} = \frac{(12 \times 12 \times 12)}{1} = 1,728$$

Example 3: Removing a negative exponent in detail

$$\begin{aligned} 5^{-3} &= \left(\frac{5}{1}\right)^{-3} && \text{Write the power as a fraction} \\ &= \left(\frac{1}{5}\right)^3 && \text{Invert the fraction; the exponent becomes positive} \\ &= \left(\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}\right) = \frac{1}{(5 \times 5 \times 5)} = \frac{1}{5^3} && \text{Equivalent forms; simplify to solve} \\ &= \frac{1}{125} = 0.008 && \text{Solution as fraction and decimal} \end{aligned}$$

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Zero Exponents

The value of a zero exponent (x^0) is always 1.

$$1 = 2^0 = 3^0 = 4^0 = 5^0 = 6^0 = 7^0 = n^0$$

Power	Factored = Solution	Pattern
2^4	$(2 \times 2 \times 2 \times 2) = 16$	
2^3	$(2 \times 2 \times 2) = 8$	$(16 \div 2)$
2^2	$(2 \times 2) = 4$	$(8 \div 2)$
2^1	$(2) = 2$	$(4 \div 2)$
2^0	$= 1$	$(2 \div 2)$
$2^{-1} = \frac{1}{2^1}$	$= \frac{1}{2}$	$(1 \div 2)$
$2^{-2} = \frac{1}{2^2}$	$\frac{1}{(2 \times 2)} = \frac{1}{4}$	$\left(\frac{1}{2} \div 2\right)$
$2^{-3} = \frac{1}{2^3}$	$\frac{1}{(2 \times 2 \times 2)} = \frac{1}{8}$	$\left(\frac{1}{4} \div 2\right)$
$2^{-4} = \frac{1}{2^4}$	$\frac{1}{(2 \times 2 \times 2 \times 2)} = \frac{1}{16}$	$\left(\frac{1}{8} \div 2\right)$

Adding Exponents

Subtracting Exponents

Multiplying Exponents

Dividing Exponents