# What is Mathematics?

Mathematics is simply a language of *pattern recognition*. We identify patterns in the world and use them to navigate the challenges of life. To do this well, we often need to use our sense of numbers.

## What are numbers?

That's a deep question, but we all already know the simple answer. A number is a word and a symbol representing an amount or a count. Let's say you walk outside your home and see two angry dogs. Even if you didn't know the word "two" or know what the corresponding number looks like, you have a good grasp of how a two-dog encounter compares with a three-, four- or zero-dog situation.

## The ability to count

From birth, we have a natural ability to count. We call this ability number sense. Studies show that while infants have no understanding of our human-made numbering systems, they can identify changes in quantity.

Number sense plays a vital role in the way animals navigate their environments—environments where objects are numerous, mobile and frequently good with Ketchup. However, our numerical sense becomes less imprecise with increasingly larger numbers. Humans, for example, are slower to compute 4 + 5 than 2 + 3.

***"There are three kinds of people in the world.***
***Them that can count, and them that can't."***

At some point in our very ancient past, we began to develop a ways to support our number sense. We started counting on our fingers and toes. This is why so many modern number systems use groups of five, 10 or 20. Base-10, or the *decimal system*, stems from using both hands to count. Base-20, or the *vigesimal system*, is based on the use of all the fingers and toes.

So ancient humans learned to externalize their number sense and, in doing so, they  created humanity's most important scientific achievement: the universal language of mathematics.

Although we have a natural ability to count, numbers can still be difficult to work with. Sure, some of us have a gift for math, but every one reaches a point where math become hard. Learning the multiplication tables is difficult because we did not evolve to handle such advanced computations as 17 x 32 = 544.

Number sense may come naturally, but developing mathematical literacy (the ability to read math) takes time and effort. Meanwhile, things just keep getting harder! Humanity's use of mathematics has steadily grown over the ages. Like science itself, math isn't one person's invention. It is a steady accumulation of knowledge throughout human history.

## The Tower of Math

Think of math as a tower. Our human height is finite (it has limits), so if we want to reach higher into the air to see farther across the landscape, we need to build something. Our natural mathematical abilities are also limited, so we have built a great tower of number systems to climb toward the stars.

# Types of Numbers

To break down the basic structure of the Tower of Math, let's first look at the raw materials. Here are the basic types of numbers:

## Integers

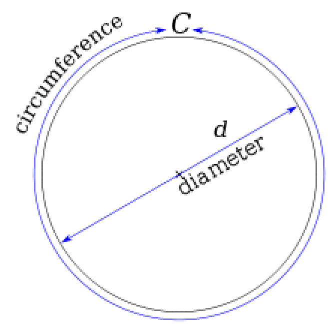
You probably know these as whole numbers, and they come in both positive and negative forms. Integers include the basic counting numbers (1-9), negative numbers (-1) and zero.

## Rational Numbers

Rational numbers include integers and simple fractions that can be expressed as a ratio of two integers. For example, 0.5 is rational because we can also write it as 1/2. Note that all integers are also fractions. For example the number 3 can be written as 3/1.

## Irrational numbers

These numbers can not be written as a ratio of two integers. Pi (the ratio of the circumference of a circle to its diameter) is a classic example. It can't be written accurately as a ratio of two integers. Here are some approximations for Pi:




/ $\pi \approx 355/113 = 3.14159292035\ldots$

/ $\pi \approx 22/7 = 3.142857142857...$

Pi is an interesting number for many reasons. You can learn a lot more about it at Wikipedia: https://en.wikipedia.org/wiki/Pi

## By the way...

***What do you get if you divide the circumference of a jack-o-lantern by its diameter?***

***Pumpkin π!***

## Real Numbers and Imaginary Numbers

As the language of mathematics grows, we develop more ways to understand numbers. Rational and irrational numbers both fall under the categories of *real numbers* and *complex numbers*. And yes, there are also *imaginary numbers* that exist outside the real number line. There are even *transcendental numbers*. We keep discovering new types of numbers, and they all become a part of the language of math.

# The Branches of Mathematics

## Arithmetic

This is the oldest and most basic form of mathematics. Arithmetic chiefly concerns the addition, subtraction, multiplication and division of real numbers that aren't negative.

## Algebra

The next level of mathematics, algebra, is arithmetic using unknown quantities. We represent the unknown parts with symbols, such as $x$ and $y$.

## Geometry

Geometry was original developed to help early humans find their way around. It deals with the measurement and properties of direction, distance and size. Geometry requires an understanding of points, lines, angles, areas and volume.

## Trigonometry

Trigonometry is used to measure triangles and the relationships between their sides and angles. While the historical origins of arithmetic, algebra and geometry are lost in the fog of ancient history, we know that trigonometry was developed the second century by the great astronomer Hipparchus of Nicaea.

## Calculus

Independently developed by both Isaac Newton and Gottfried Leibniz in the 17th century, Calculus deals with the calculation of instantaneous rates of change. It focuses on finding the kinds of answers that may be very close to zero or infinity.

## Invented or discovered?

The tower of mathematics has enabled human culture to rise and flourish, to speak across time and space, and to understand both the inner mysteries of life to the outer mysteries of our world. But did we truly build this tower out of our own ingenuity?

***How do mathematics teachers scold students?***
***"If I've told you n times, I've told you n+1 times..."***

## Questions

1. What is your natural mathematical ability? For example, how many things can you instantly sense without needing to think or count?
2. What is the difference between arithmetic and algebra?
3. Was mathematics invented or discovered?
4. How do we know when our mathematics is accurate?
5. Is mathematics always true? Do you think there might be a place or time where our mathematics is “wrong?” What would that place be like?