The Math Revolution

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The number of American teens who excel at advanced math has surged. Why?

On a sultry evening last July, a tall, soft-spoken 17-year-old named David Stoner and nearly 600 other math whizzes from all over the world sat huddled in small groups around wicker bistro tables, talking in low voices and obsessively refreshing the browsers on their laptops. The air in the cavernous lobby of the Lotus Hotel Pang Suan Kaew in Chiang Mai, Thailand, was humid, recalls Stoner, whose light South Carolina accent warms his carefully chosen words. The tension in the room made it seem especially heavy, like the atmosphere at a high-stakes poker tournament.

Stoner and five teammates were representing the United States in the 56th International Mathematical Olympiad. They figured they'd done pretty well over the two days of competition. God knows, they'd trained hard. Stoner, like his teammates, had endured a grueling regime for more than a year—practicing tricky problems over breakfast before school and taking on more problems late into the evening after he completed the homework for his college-level math classes. Sometimes, he sketched out proofs on the large dry-erase board his dad had installed in his bedroom. Most nights, he put himself to sleep reading books like New Problems in Euclidean Geometry and An Introduction to Diophantine Equations.

Still, it was hard to know how his team had stacked up against those from the perennial powers China, Russia, and South Korea. "I mean, the gold? Did we do well enough to get the gold?" he said. "At that moment, it was hard to say." Suddenly, there was a shout from a team across the lobby, then a collective intake of breath as the Olympians surged closer to their laptops. As Stoner tried to absorb what he saw on his own computer screen, the noise level in the lobby grew from a buzz to a cheer. Then one of his team members gave a whoop that ended in the chant "U.S.A.! U.S.A.!," and the smattering of applause from the other Olympians grew more robust, and finally thunderous. Beaming, one of Stoner's teammates pulled a small American flag out of his backpack and began waving it. Stoner was grinning. For the first time in 21 years, the United States team had won first place. Speaking last fall from his dorm at Harvard, where he is now a freshman, Stoner recalled his team's triumph with quiet satisfaction. "It was a really great moment. Really great. Especially if you love math."

It also wasn't an aberration. You wouldn't see it in most classrooms, you wouldn't know it by looking at slumping national test-score averages, but a cadre of American teenagers are reaching world-class heights in math—more of them, more regularly, than ever before. The phenomenon extends well beyond the handful of hopefuls for the Math Olympiad. The students are being produced by a new pedagogical ecosystem—almost entirely extracurricular—that has developed online and in the country's rich coastal cities and tech meccas. In these places, accelerated students are learning more and learning faster than they were 10 years ago—tackling more-complex material than many people in the advanced-math community had thought possible. "The bench of American teens who can do world-class math," says Po-Shen Loh, the head coach of the U.S. team, "is significantly wider and stronger than it used to be."

The change is palpable at the most competitive colleges. At a time when calls for a kind of academic disarmament have begun echoing through affluent communities around the nation, a faction of students are moving in exactly the opposite direction. "More freshmen arrive at elite colleges with exposure to math topics well outside of what has traditionally been taught in American high schools," says Loh. "For American students who have an appetite to learn math at a high level," says Paul Zeitz, a mathematics professor at the University of San Francisco, "something very big is happening. It's very dramatic and it's happening very fast."

In the past, a small number of high-school students might have attended rigorous and highly selective national summer math camps like Hampshire College's Summer Studies in Mathematics, in Massachusetts, or the Ross Mathematics Program at Ohio State, both of which have been around for decades. But lately, dozens of new math-enrichment camps with names like MathPath, AwesomeMath, MathILy, Idea Math, sparc, Math Zoom, and Epsilon Camp have popped up, opening the gates more widely to kids who have aptitude and enthusiasm for math, but aren't necessarily prodigies. In Silicon Valley and the Bay Area, math circles—some run by tiny nonprofit organizations or a single professor, and offering small groups of middle- and high-school math buffs a chance to tackle problems under the guidance of graduate students, teachers, professors, engineers, and software designers—now have long wait lists. In New York City last fall, it was easier to get a ticket to the hit musical Hamilton than to enroll your child in certain math circles. Some circles in the 350-student New York Math Circle program run out of New York University filled up in about five hours.*

Math competitions are growing in number and popularity too. The number of U.S. participants in Math Kangaroo, an international contest for first- through 12th-graders that came to American shores in 1998, grew from 2,576 in 2009 to 21,059 in 2015. More than 10,000 middle- and high-school students haunt chat rooms, buy textbooks, and take classes on the advanced-math learners' Web site the Art of Problem Solving. This fall, the Art of Problem Solving's founder, Richard Rusczyk, a former Math Olympian who left his job in finance 18 years ago, will open two brick-and-mortar centers in the Raleigh, North Carolina, and Rockville, Maryland, areas, with a focus on advanced math. An online program for elementary-school students will follow. Last fall, Zeitz—along with another math professor, a teacher, and a private-equity manager—opened the Proof School, a small independent secondary school in San Francisco similarly centered on amped-up math. Before the inaugural school year even began, school officials were fielding inquiries from parents wondering when a Proof School would be opening on the East Coast and whether they could get their child on a waiting list. "The appetite among families for this kind of math instruction," Rusczyk says, "seems boundless."

Parents of students in the accelerated-math community, many of whom make their living in stem fields, have enrolled their children in one or more of these programs to supplement or replace what they see as the shallow and often confused math instruction offered by public schools, especially during the late-elementary and middle-school years. They have reason to do so. According to the Bureau of Labor Statistics, much of the growth in our domestic economy will come from stem-related jobs, some of which are extremely well paid. College freshmen have heard that message; the number who say they want to major in a stem field is up. But attrition rates are very high: Between 2003 and 2009, 48 percent of students pursuing a bachelor's degree in a stem field switched to another major or dropped out—many found they simply didn't have the quantitative background they needed to succeed.

The roots of this failure can usually be traced back to second or third grade, says Inessa Rifkin, a co-founder of the Russian School of Mathematics, which this year enrolled 17,500 students in after-school and weekend math academies in 31 locations around the United States. In those grades, many education experts lament, instruction—even at the best schools—is provided by poorly trained teachers who are themselves uncomfortable with math. In 1997, Rifkin, who once worked as a mechanical engineer in the Soviet Union, saw this firsthand. Her children, who attended public school in affluent Newton, Massachusetts, were being taught to solve problems by memorizing rules and then following them like steps in a recipe, without understanding the bigger picture. "I'd look over their homework, and what I was seeing, it didn't look like they were being taught math," recalls Rifkin, who speaks emphatically, with a heavy Russian accent. "I'd say to my children, 'Forget the rules! Just think!' And they'd say, 'That's not how they teach it here. That's not what the teacher wants us to do.'" That year, she and Irina Khavinson, a gifted math teacher she knew, founded the Russian School around her dining-room table.

Teachers at the Russian School help students achieve fluency in arithmetic, the fundamentals of algebra and geometry, and later, higher-order math. At every level, and with increasing intensity as they get older, students are required to think their way through logic problems that can be resolved only with creative use of the math they've learned.

One chilly December Sunday at a school in Bensonhurst, Brooklyn, seven second-graders filed past a glossy poster showing Russian School students who had recently medaled in math competitions. They settled into their seats as their teacher, Irine Rober, showed them conceptual examples of addition and subtraction by ripping paper in half and by adding weights to each side of a scale to balance it. Simple stuff. Then the students took turns coming to the blackboard to explain how they'd used addition and subtraction to solve an equation for x, which required a bit more thinking. After a brief break, Rober asked each child to come up with a narrative that explained what the expression 49+(18–3) means. The children invented stories involving fruit, the shedding and growing of teeth, and, to the amusement of all, toilet monsters.

Although the students were laughing, there was nothing superficial or perfunctory about their explanations. Rober and her class listened carefully to the logic embedded in each of the stories. When one young boy, Shawn, got tangled up in his reasoning, Rober was quick to point to the exact spot where his thinking went awry (in the enthusiastic telling of a tale about farmers, bountiful harvests, and apple-eating varmints, Shawn began by talking about what happened to the 49 apples, when the order of operations demanded that he first describe a reduction in the 18 apples). Rober gently set him straight. Later, the children told stories about 49–(18+3) and 49–(18-3) too.

Rifkin trains her teachers to expect challenging questions from students at every level, even from pupils as young as 5, so lessons toggle back and forth between the obvious and the mind-bendingly abstract. "The youngest ones, very naturally, their minds see math differently," she told me. "It is common that they can ask

simple questions and then, in the next minute, a very complicated one. But if the teacher doesn't know enough mathematics, she will answer the simple question and shut down the other, more difficult one. We want children to ask difficult questions, to engage so it is not boring, to be able to do algebra at an early age, sure, but also to see it for what it is: a tool for critical thinking. If their teachers can't help them do this, well—" Rifkin searched for the word that expressed her level of dismay. "It is a betrayal."

For a subject that has been around almost as long as civilization itself, there remains a surprising degree of contention among experts about how best to teach math. Fiery battles have been waged for decades over what gets taught, in what order, why, and how. Broadly speaking, there have been two opposing camps. On one side are those who favor conceptual knowledge—understanding how math relates to the world—over rote memorization and what they call "drill and kill." (Some well-respected math-instruction gurus say that memorizing anything in math is counterproductive and stifles the love of learning.) On the other side are those who say memorization of multiplication tables and the like is necessary for efficient computation. They say teaching students the rules and procedures that govern math forms the bedrock of good instruction and sophisticated mathematical thinking. They bristle at the phrase drill and kill and prefer to call it simply "practice."

The Common Core State Standards Initiative walks a narrow path through that minefield, calling for teachers to place equal importance on "mathematical understanding" and "procedural skills." It's too early to know what effect the initiative will have. To be sure, though, most students today aren't learning much math: Only 40 percent of fourth-graders and 33 percent of eighth-graders are considered at least "proficient." On an internationally administered test in 2012, just 9 percent of 15-year-olds in the United States were rated "high scorers" in math, compared with 16 percent in Canada, 17 percent in Germany, 21 percent in Switzerland, 31 percent in South Korea, and 40 percent in Singapore.

The new outside-of-school math programs like the Russian School vary in their curricula and teaching methods, but they have key elements in common. Perhaps the most salient is the emphasis on teaching students to think about math conceptually and then use that conceptual knowledge as a tool to predict, explore, and explain the world around them. There is a dearth of rote learning and not much time spent applying a list of memorized formulas. Computational speed is not a virtue. ("Cram schools," featuring a mechanistic, test-prep approach to learning math, have become common in some immigrant communities, and plenty of tutors of affluent children use this approach as well, but it is the opposite of what's taught in this new type of accelerated-learning program.) To keep pace with their classmates, students quickly learn their math facts and formulas, but that is more a by-product than the point.

The pedagogical strategy at the heart of the classes is loosely referred to as "problem solving," a pedestrian term that undersells just how different this approach to math can be. The problem-solving approach has long been a staple of math education in the countries of the former Soviet Union and at elite colleges such as MIT and Cal Tech. It works like this: Instructors present small clusters of students, usually grouped by ability, with a small number of open-ended, multifaceted situations that can be solved by using different approaches.

Here's an example from the nascent math-and-science site Expii.com:

Imagine a rope that runs completely around the Earth's equator, flat against the ground (assume the Earth is a perfect sphere, without any mountains or valleys). You cut the rope and tie in another piece of rope that is 710 inches long, or just under 60 feet. That increases the total length of the rope by a bit more than the length of a bus, or the height of a 5-story building. Now imagine that the rope is lifted at all points simultaneously, so that it floats above the Earth at the same height all along its length. What is the largest thing that could fit underneath the rope?

The options given are bacteria, a ladybug, a dog, Einstein, a giraffe, or a space shuttle. The instructor then coaches all the students as they reason their way through. Unlike most math classes, where teachers struggle to impart knowledge to students—who must passively absorb it and then regurgitate it on a test—problem-solving classes demand that the pupils execute the cognitive bench press: investigating, conjecturing, predicting, analyzing, and finally verifying their own mathematical strategy. The point is not to accurately execute algorithms, although there is, of course, a right answer (Einstein, in the problem above). Truly thinking the problem through—creatively applying what you know about math and puzzling out possible solutions—is more important. Sitting in a regular ninth-grade algebra class versus observing a middle-school problem-solving class is like watching kids get lectured on the basics of musical notation versus hearing them sing an aria from Tosca.

In my experience, a common emotion at New York Math Circle, at the Russian School, in the chat rooms of the Art of Problem Solving and similar Web site, is authentic excitement—among the students, but also among the

teachers—about the subject itself. Even in the very early grades, instructors tend to be deeply knowledgeable and passionately engaged. "Many of them are working in the fields that use math—chemistry, meteorology, and engineering—and teach part-time," Rifkin says. They are people who themselves find the subject approachable and deeply interesting, and they are encouraged to convey that.

But excitement aside, the pedagogy is very deliberate. At the Russian School, lessons are carefully structured and each teacher's lesson plan is reviewed and revised by a mentor. Instructors watch videos of master teachers deftly helping to clear up students' misunderstandings of particular concepts. Teachers gather by videoconference to critique one another's instructional technique.

Many of these programs—especially the camps, competitions, and math circles—create a unique culture and a strong sense of belonging for students who have a zest for the subject but all the awkwardness and uneven development of the typical adolescent. "When I attended my first math competition," at age 11, "I understood for the first time that my tribe was out there," said David Stoner, who joined a math circle a year later, and soon thereafter became a habitué of the Art of Problem Solving. Freewheeling collaboration across age, gender, and geography is a baseline value. Although the accelerated-math community has historically been largely male, girls are getting involved in increasing numbers, and making their presence felt. Kids blow off steam by playing strategy board games like Dominion and Settlers of Catan, or "bug house" chess, a high-speed, multiboard variation of the old standby. Insider humor abounds. A typical T-shirt slogan: $\sqrt{-1} 2^3 \sum \pi \dots$ and it was delicious! (Translation: "I ate some pie ...") At the Math Olympiad Summer Program, a training ground for future Olympians, one of the acts in the talent show last June involved a group of youngsters developing computer code while holding a plank pose.

The students speak about career ambitions with a rare degree of assurance. Problem-solving for fun, they know, leads to problem-solving for profit. The link can be very direct: Some of the most recognizable companies in the tech industry regularly prospect, for instance, on Brilliant.org, an advanced-math-community Web site launched in San Francisco in 2012. "Money follows math" is a common refrain.

Although efforts are under way on many fronts to improve math education in public schools using some of the techniques found in these enriched classes, measurable gains in learning have proved elusive.

Nearly everyone in the accelerated-math community says that the push to cultivate sophisticated math minds needs to start early and encompass plenty of thoughtful, conceptual learning experiences in elementary and middle school. The proportion of American students who can do math at a very high level could be much larger than it is today. "Will they all learn it at the same rate? No, they will not," says Loh, the U.S. math team's head coach. "But I assure you that with the right instruction and steady effort, many, many more American students could get there."

Students who show an inclination toward math need additional math opportunities—and a chance to be around other math enthusiasts—in the same way that a kid adept with a soccer ball might eventually need to join a traveling team. And earlier is better than later: The subject is relentlessly sequential and hierarchical. "If you wait until high school to attempt to produce accelerated math learners," Loh told me, "the latecomers will find themselves missing too much foundational thinking and will struggle, with only four short years before college, to catch up." These days, it is a rare student who can move from being "good at math" in a regular public high school to finding a place in the advanced-math community.

All of which creates a formidable barrier. Most middle-class parents might research sports programs and summer camps for their 8- and 9-year-old children, but would rarely think of supplemental math unless their kid is struggling. "You have to know about these programs, live in a neighborhood that has these resources, or at least know where to look," says Sue Khim, a co-founder of Brilliant.org. And since many of the programs are private, they are well out of reach for the poor. (A semester in a math circle can cost about \$300, a year at a Russian School up to \$3,000, and four weeks in a residential math program perhaps twice that.) National achievement data reflect this access gap in math instruction all too clearly. The ratio of rich math whizzes to poor ones is 3 to 1 in South Korea and 3.7 to 1 in Canada, to take two representative developed countries. In the U.S., it is 8 to 1. And while the proportion of American students scoring at advanced levels in math is rising, those gains are almost entirely limited to the children of the highly educated, and largely exclude the children of the poor. By the end of high school, the percentage of low-income advanced-math learners rounds to zero.

To Daniel Zaharopol, the founder and executive director of Bridge to Enter Advanced Mathematics (beam), a nonprofit organization based in New York City, the short-term solution is logical. "We know that math ability is universal and interest in math is spread pretty much equally through the population," he says, "and we see there are almost no low-income, high-performing math students. So we know that there are many, many students who

have the potential for high achievement in math but who have not had opportunity to develop their math minds, simply because they were born to the wrong parents or in the wrong zip code. We want to find them."

In an experiment that is being closely watched by educators and members of the advanced-math community, Zaharopol, who majored in math at MIT before getting a master's in math and teaching math, spends each spring visiting middle schools in New York City that serve low-income kids. He is prospecting for students who, with the right instruction and some support, can take their place, if not at the International Math Olympiad, then at a less selective competition, and in a math circle, and eventually at a stem program at a competitive college.

Zaharopol doesn't look for the best all-around students to admit to his program, which provides the kind of comprehensive support that wealthy math nerds get: a three-week residential math camp the summer before eighth grade, enhanced instruction after school, help with applying to math circles, and coaching for math competitions, as well as basic advice on high-school selection and college applications. Those who get perfect grades in math are interesting to him, but only to a point. "They don't have to like school or even like math class," he says. Instead, he is looking for kids with a confluence of specific abilities: strong reasoning, lucid communication, stamina. A fourth, more ineffable quality is crucial: "I look for kids who take pleasure in resolving complicated problems," Zaharopol says. "Actually doing math should bring them joy."

Five years ago, when Zaharopol entered M.S. 343, a boxy-looking building in a rough section of the South Bronx, and sat down with a seventh-grader, Zavier Jenkins, who had a big smile and a Mohawk, nothing about the setup was auspicious. With just 13 percent of kids performing at grade level in English and 57 percent in math, M.S. 343 seemed an unlikely incubator for tomorrow's tech mogul or medical engineer.

But in a quiet conversation, Zaharopol learned that Jenkins had what his siblings and peers considered a quirky affinity for patterns and an inclination toward numbers. Lately, Jenkins confided to Zaharopol, a certain frustration had set in. He could complete his math assignments accurately, but he was growing bored.

Zaharopol asked Jenkins to do some simple computations, which he handled with ease. Then Zaharopol threw a puzzle at Jenkins and waited to see what would happen:

You have a drawer full of socks, each one of which is red, white, or blue. You start taking socks out without looking at them. How many socks do you need to take out of the drawer to be sure you have taken out at least two socks that are the same color?

"For the first time, I was presented with a math problem that didn't have an easy answer," Jenkins recalls. At first, he simply multiplied two by three to get six socks. Dissatisfied, he began sifting through other strategies.

"I was very encouraged by that," Zaharopol told me. "Many kids just assume they have the right answer." After a few minutes, he offered to show Jenkins one way to reason through the problem. The energy in the room changed. "Not only did Zavier come up with the right answer"—four—"but he really understood it very thoroughly," Zaharopol said. "And he seemed to take delight in the experience." Four months later, Jenkins was living with 16 other rising eighth-graders in a dorm at the beam summer program on Bard College's campus in upstate New York, being coached on number theory, recursion, and graph theory by math majors, math teachers, and math professors from top universities around the country. With some counseling from beam, he entered a coding program, which led to an internship at Microsoft. Now a high-school senior, he has applied to some of the top engineering schools in the country.

Beam, which is five years old, has already quadrupled in size—it hosted 80 middle-school students at its summer program last year and has about 250 low-income, high-performing students in its network. But its funding remains limited. "We know there are many, many more low-income kids who we don't reach and who simply don't have access to these programs," Zaharopol said.

There is already a name for the kind of initiative that might, in part, bring the benefits of beam, math circles, the Russian School, or the Art of Problem Solving to a broader array of students, including middle- and low-income ones: gifted-and-talented programs, which are publicly funded and can start in elementary school. But the history of these programs is fraught. Admission criteria vary, but they have tended to favor affluent children. Teachers can be lobbied for a recommendation; some standardized entry tests measure vocabulary and general knowledge, not creative reasoning. In some places, parents pay for their children to be tutored for the admission exam, or even privately tested to get in.

As a result, while many such programs still exist, they've been increasingly spurned by equity-minded school administrators and policy makers who see them as a means by which predominately affluent white and Asian

parents have funneled scarce public dollars toward additional enrichment for their already enriched children. (The vaguely obnoxious label itself—"gifted and talented"—hasn't helped matters.)

Children should see math "for what it is: a tool for critical thinking. If their teachers can't help them do this, well—it is a betrayal."

The No Child Left Behind Act, which shaped education for nearly 15 years, further contributed to the neglect of these programs. Ignoring kids who may have had aptitude or interest in accelerated learning, it demanded that states turn their attention to getting struggling learners to perform adequately—a noble goal. But as a result, for years many educators in schools in poor neighborhoods, laser-focused on the low-achieving kids, dismissed suggestions that the minds of their brightest kids were lying fallow. Some denied that their schools had any gifted children at all.

The cumulative effect of these actions, perversely, has been to push accelerated learning outside public schools—to privatize it, focusing it even more tightly on children whose parents have the money and wherewithal to take advantage. In no subject is that clearer today than in math.

The good news is that education policy may be beginning to swing back. Federal and state legislators increasingly seem to agree that all teenagers could benefit from the kind of accelerated-learning opportunities once reserved for high-aptitude kids in affluent neighborhoods, and many public high schools have been pushed to offer more Advanced Placement classes and to expand enrollment in online college courses. But for many middle- and low-income students who might have learned to love math, those opportunities come too late.

Perhaps it is a hopeful sign, then, that the newly authorized Every Student Succeeds Act, which recently replaced No Child Left Behind, asks states to recognize that such students can exist in every precinct, and to track their progress. For the first time in the nation's history, the law also explicitly allows schools to use federal dollars to experiment with ways of screening for low-income, high-ability students in the early years and to train teachers to serve them. Universal screening in elementary school might be a good start. From 2005 to 2007, school officials in Broward County, Florida, concerned that poor kids and English-language learners were being under-referred to gifted programs, gave all second-graders, rich and poor, a nonverbal reasoning test, and the high scorers an IQ test. The criteria for "gifted" status weren't weakened, but the number of disadvantaged children identified as having the capacity for accelerated learning rose 180 percent.

Whether individual states take up this challenge, and do so effectively, is their decision, but advocates say they are mounting a campaign to get started. Perhaps the moment is right for members of the advanced-math community, who have been so successful in developing young math minds, to step in and show more educators how it could be done.